PROBLEMS.

48. Proposed by SETH PRATT, C. E., Assyria, Michigan.

What is the interest of $500 for 10 years at 10% per annum, when the intervals of compounding are infinitely small?

49. Proposed by P. S. BERG, Apple Creek, Ohio.

A man having lent $6000 at 6% interest. payable quarterly. wishes to receive his interest in equal proportions monthly, and in advance; how much ought he to receive each month?

GEOMETRY.

Conducted by B.F.FINKEL, Kidder, Missouri. All Contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

36. Proposed by O. W. ANTHONY, Mexico, Missouri.

From two points, one on each of the opposite sides of a parallelogram, lines are drawn to the opposite vertices. Through the points of intersection of these lines a line is drawn. Prove that it divides the parallelogram into two equal parts.

II. Solution by LEONARD E. DICKSON, M. A., Fellow in Mathematics, University of Chicago.

The line in question IK will cut the parallelogram into two equal parts if it is proved to pass through O, the intersection of the diagonals. The latter theorem is true for any quadrilateral (Pappus, *Mathematicae Collectiones*, VII, p. 139).

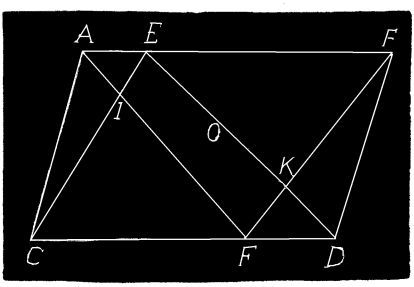


Fig. 1.

If a hexagon $AB'CA'BC'$ has its summits of even order upon one straight line and those of odd order upon another, the three pairs of opposite sides (AB' and $A'B$, BC' and $B'C$, CA' and $C'A$) intersect in three points on a straight line. See Cremona for proof. But even this is only Pascal's theorem for a hexagon inscribed in a conic, when the latter degenerates to a pair of straight lines.

Analytical proof for parallelogram:

Take as axes lines ∥ to AB and BD through O. Let $AB=2b$, $BD=2a$. Call $I(x',y')$ and $K(x_1, y_1)$. Equation to AF is $(x+b)(y'-a)=(y-a)(x'+b)$. Equation to DE is $(x+b)(y'+a)=(y+a)(x'+b)$. Hence the co-ordinates of F are $\left(\frac{2ax'+by'+ab}{a-y'}, -a\right)$; of E, $\left(\frac{2ax'-by'+ab}{a+y'}, a\right)$.

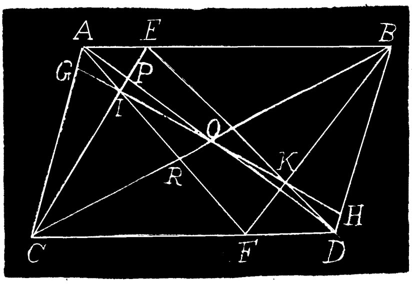
Hence, equation to BF is $a(x-b)(y'-a)=(y-a)(ax'+by')$; equation to ED is $a(x-b)(y'+a)=(y+a)(ax'-by')$.

$\therefore$ the co-ordinates of K are $x_1=\frac{a^2x'(x'+b)}{a^2x'+by'^2}$; $y_1=\frac{a^2y'(x'+b)}{a^2x'+by'^2}$.

$\therefore$ $\frac{x_1}{y_1}=\frac{x'}{y'}$, or IK passes through the origin O

III. **Solution by J. K. ELLWOOD, A. M., Principal Colfax School, Pittsburg, Pennsylvania.**

Let E and F be the points taken in opposite sides of the parallelogram $ABCD$, GH the line drawn through the points of intersection.



FIRST.—The intersection, O, of the diagonals of $\square ABCD$ is in the line GH.

The △'s CIF and AIE are similar, as are FKD and EKB, and AOB and COD. Hence

$BO:OC::AB:CD$; or $BO\times CD=OC\times AB$....(1).

$FK:BK::FD:EB$; or $FK\times EB=BK\times FD$....(2).

$CI:EI::FI:AI$; by composition,

$CI:CE::FI:AF$; or $CI\times AF=CE\times FI$....(3).

The sides produced of the △'s CIF, AIE, CIR are cut by the lines DPA, COB, and OA, respectively.

$\therefore$ $CP\times FD\times AI=CD\times AF\times IP$*....(4).

$IR\times AB\times CE=AR\times EB\times CI$....(5).

$CO\times IP\times AR=CP\times AI\times OR$....(6).

Multiplying equations (1),(2),(3),(4),(5),(6) together, we have

$BO\times FK\times IR=BK\times FI\times OR$....(7).

$\therefore$ (By appended proof **) O is in the line $GIKH$.

SECOND.—In the similar △'s AOG and HOD, $AO=OD$.

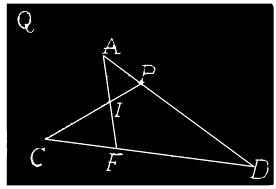
$\therefore$ $HD=AG$. $\therefore$ $GC=HB$, and $AG+HB=DH+CG$.

Area trapezoid $AGHB=\frac{1}{2}(AG+HB)\times$alt.

Area trapezoid $GCDH=\frac{1}{2}(DH+CG)\times$alt.

Then, since $AG+HB=DH+CG$, these areas are equal, and the line GH divides $ABCD$ into two equal parts. *Q. E. D.*

EXPLANATORY PROOF. Let CFI (see fig. 1.) be any $\triangle$, and produce the sides to D, P, A in any *straight line*, as DA.



Through C draw CQ parallel to FA and meeting DA, produced, in Q. The $\triangle$'s DAF and DQC are similar, as are API and QPC.

Hence $FD:AF::CD:CQ$, and

$$AI:IP::CQ:CP.$$

$$\therefore \quad FD\times AI:AF\times IP::CD:CP.$$

$$\therefore \quad FD\times AI\times CP=AF\times IP\times CD.$$

This is equation (4)a bove; (5) and (6) may be similarly obtained.

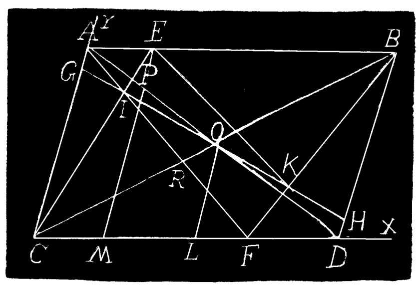
It may be shown in a similar manner that, if a *straight line* cuts two sides and the third side produced of a $\triangle$, the product of any three of the non-adjacent segments (of the sides) is equal to the product of the other three segments. The produced side is one segment, the prolongment another.

** CONVERSELY. If three points divide the two sides and determine the prolongment of the third side produced so that the product of any three non-adjacent segments shall be equal to the product of the other three. then are these points in the same *straight line*.

Equation (7) is derived from the $\triangle$ BFR. The points K, O are in the *sides*, and I is in RF produced. Since in equation (7) the product of three non-adjacent segments is equal to the product of the other three, the three points I, O, K are in the straight line GH.

IV. Solution by Professor G. B. M. ZERR, A. M., Principal of High School, Staunton, Virginia.

Let E, F be the points. I, K the intersection of ED, FB and EC, FA, O the intersection of the diagonals BC, AD. Let $CD=AB=a$, $AC=ME=BD=b$, $CM=c$, $CF=d$, $h=$the perpendicular from A on BD.



Then $CL=\frac{1}{2}a$, $LO=\frac{1}{2}b$,

$\frac{y}{x}=\frac{b}{c}$, equation to CE; $\frac{x}{d}+\frac{y}{b}=1$, equation to AF. $y=\frac{b(a-x)}{a-c}$, equation to DE, $y=\frac{b(x-d)}{a-d}$, equation to BF.

$\left(\frac{cd}{c+d}, \frac{bd}{c+d}\right)$ co-ordinates of I, $\left(\frac{a^2-dc}{2a-c-d}, \frac{b(a-d)}{2a-c-d}\right)$ co-ordinates of K.

$(2cd-ac-ad)y-(bd-bc)x=bd(c-a)$, the equation to IO.

This line cuts DE at the point $\left(\frac{a^2-dc}{2a-c-d}, \frac{b(a-d)}{2a-c-d}\right)$

$\therefore$ IO passes through K, and IO and GH are the same line.

In the triangle GOA and HKD, $AO=OD$, $\angle AOG=\angle DOH$,

$\angle OAG = \angle ODH$. $\therefore GA = DH$. Similarly $CG = BH$.

$\therefore AG + BH = CG + DH$.

$\therefore \frac{1}{2}h(AG+BH) = \frac{1}{2}h(CG+DH)$.

$\therefore$ Area $AGHD$ = area $CGHD$.

Good solutions of this problem were received from *Professors Wm. Symmonds, and Cooper D. Schmitt.*

This problem has proved to be a very interesting one and for that reason we have given it extra space. The proposition to which Prof. Ellwood has given a proof explanatory to the proposition under consideration is known as the proposition of *Menelaus*. See *Halsted's Elementary Synthetic Geometry*, p. 117. Editor.

PROBLEMS.

42. Proposed by G. I. HOPKINS, Instructor in Mathematics and Physics in High School, Manchester, New Hampshire.

If the bisectors of two angles of a triangle are equal the triangle is isosceles.

[The term *bisector* in this theorem means the line which divides an angle into two equal parts and terminates in the opposite side.]

43. Proposed by J. F. W. SCHEFFER, Hagerstown, Maryland.

The consecutive sides of a quadrilateral are a, b, c, d. Supposing its diagonals to be equal, find them and also the area of the quadrilateral.

CALCULUS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

27. Proposed by Professor G. B. M. ZERR, A. M., Principal of High School. Staunton, Virginia.

A runs around the circumference of a circular field with velocity m feet: B starts from the centre with velocity $n > m$ feet to catch A. The straight line joining their positions always passes through the centre. Find the equation to the curve described by B, the distance he runs and the time occupied.

I. Solution by A. H. HOLMES, Brunswick, Maine, H. W. DRAUGHON, Olio. Mississippi, and the PROPOSER.

Let A be the point of starting of the pursued, P, B, the position of the pursuer and pursued at any time.

Let $OA = a$, $OP = r$, $\angle BOA = \theta$, $\frac{m}{n} = u$, arc $OP = s$.